MATH 1A - MOCK FINAL

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Name: ____________________

Instructions: This is a mock final, designed to give you an idea of what the actual final will look like. Make sure you do it, the actual exam will be very similar to this one (in length and in difficulty)!

1		20
2		10
3		50
4		20
5		20
6		20
7		10
Bonus 1		5
Bonus 2		5
Bonus 3		5
Total		150

Date: Friday, August 5th, 2011.

1. *(20 points)* Use the **definition** of the integral to evaluate:

$$\int_1^2 x^2 dx$$

You may use the following formulas:

$$\sum_{i=1}^{n} 1 = n \qquad \sum_{i=1}^{n} i = \frac{n(n+1)}{2} \qquad \sum_{i=1}^{n} i = \frac{n(n+1)(2n+1)}{6} \qquad \sum_{i=1}^{n} i^3 = \frac{n^2(n+1)^2}{4}$$

2. *(10 points)* Evaluate the following limit:

$$\lim_{n\to\infty} \frac{1}{n}\left(\sqrt{\frac{1}{n}}+\sqrt{\frac{2}{n}}+\cdots+\sqrt{\frac{n}{n}}\right)$$

3. *(50 points, 5 points each)* Find the following:

(a) The antiderivative F of $f(x) = x^2 + 3x^3 - 4x^7$ which satisfies $F(0) = 1$

(b) $\int_{-1}^{1} |x| \, dx$ (**Hint:** Draw a picture)

(c) $\int_{-\pi}^{\pi} \sin(x)(1 + \cos(x) + e^{x^2} + 42x^{2012})dx$

(d) $\int x^2 + 1 + \frac{1}{x^2+1} dx$

(e) $\int_1^e \frac{(\ln(x))^2}{x} dx$

(f) $\int_{\pi}^{2\pi} (\cos(x) - 2\sin(x))\, dx$

(g) $g'(x)$, where $g(x) = \int_x^{e^x} \sqrt{1+t^2}dt$

(h) $\int_0^{\frac{\pi}{4}} \frac{1+\cos^2(\theta)}{\cos^2(\theta)}d\theta$

(i) $\int e^x \sqrt{1+e^x} dx$

(j) The average value of $f(x) = \sin(x)$ on $[-\pi, \pi]$

4. *(20 points)* Find the area of the region enclosed by the curves:

$$y = x^2 - 4 \quad \text{and} \quad y = 4 - x^2$$

5. *(20 points, 5 points each)* Find the following limits

(a) $\lim_{x\to\infty} \sqrt{x^2+x} - x$

(b) $\lim_{x\to\infty} (1+x)^{\frac{1}{x}}$

(c) $\lim_{x\to 0} xe^{\sin(\frac{1}{x})}$

(d) $\lim_{x\to\infty} \frac{(\ln(x))^2}{x}$

6. *(20 points, 5 points each)* Find the derivatives of the following functions
   (a) $f(x) = \sin(x)e^{\tan(x)}$

   (b) $f(x) = x^{\cos(x)}$

(c) y', where $x^3 + y^3 = xy$

(d) y' at $(0, 1)$, where $\frac{x^2+y^2}{x^2-y^2} = -y$

7. *(10 points)* Find the absolute maximum and minimum of the following function on $[0, \pi]$:

$$f(x) = x + \cos(x)$$

Bonus 1 *(5 points)* Show that if f is continuous on $[0, 1]$, then $\int_0^1 f(x)dx$ is bounded, that is, there are numbers m and M such that:

$$m \leq \int_0^1 f(x)dx \leq M$$

Hint: Use one of the 'value' theorems that we haven't used a lot in this course (see section 4.1)

Bonus 2 *(5 points)* If $f(x) = Ax^3 + Bx^2 + Cx + D$ is a polynomial such that:

$$\frac{A}{4} + \frac{B}{3} + \frac{C}{2} + D = 0$$

Show that f has at least one zero in $(0, 1)$.

Hint: What is the *average* value of f on $[0, 1]$?

Bonus 3 *(5 points)* Another way to define $\ln(x)$ is:

$$\ln(x) = \int_1^x \frac{1}{t} dt$$

Show **using this definition only** that for all a and b:

$$\ln(ab) = \ln(a) + \ln(b)$$

Hint: Fix a constant a, and consider the function:

$$g(x) = \ln(ax) - \ln(x) - \ln(a)$$